



Math 123 Final Tutorial

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Derivative Warmups

$$y = (\ln(x))^{\sqrt{x}}$$

$$f(x) = (\tan x)^{\cos x}$$

$$y = \sqrt{\sin^{-1} x^2}$$

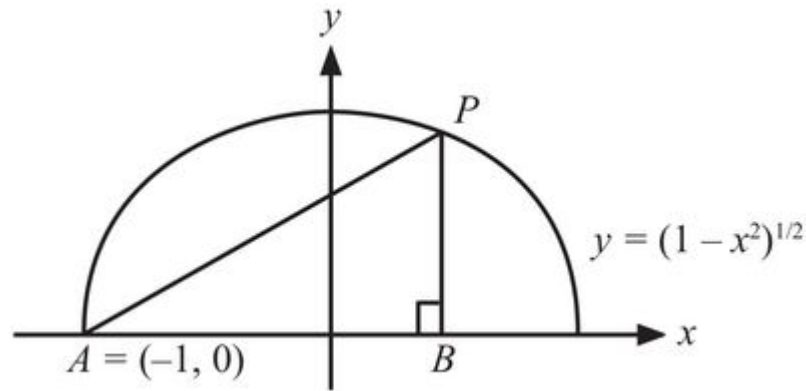


Optimization (extreme-value problems) - 1

Find the area of the largest rectangle which has two vertices on the x -axis and two vertices lie on the graph of the function $y = 8 - x^2$ with $-\sqrt{8} \leq x \leq \sqrt{8}$

Optimization (extreme-value problems) - 2

Find the coordinates of the point P , which lies on the curve , $y = \sqrt{1 - x^2}$ in the diagram below for which the area of the right triangle ABP is the maximum





Limits - 1

Evaluate

$$\lim_{x \rightarrow \infty} \left(1 + \frac{5}{x} \right)^x$$



Limits - 2

Evaluate

$$\lim_{x \rightarrow 0^+} x^x$$



Related Rates - 1

A light sits atop a post H meters high, and a person of height h , where $H > h$, walks along a straight line away from the lamppost at a speed of v m/s. At what rate is the person's shadow lengthening?



Related Rates - 2

A vertical cylindrical tank with radius $3m$ is being filled with water at a rate of $5m^3/min$.
How fast is the height of the water increasing?



Antiderivatives

$$\int k dx = kx + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int x^n dx : (n \neq -1) = \frac{x^{n+1}}{n+1} + c$$



Antiderivatives

Find antiderivatives for the following functions

$$f(x) = 5\sqrt{x} - 7x$$

$$f(x) = \frac{x^3 + x^2 - x + 1}{2x^2}$$

$$f(x) = \frac{3}{2 + 2x^2}$$



First Order Separable DE and IVP

Solve

$$y'' = \sin x, y(\pi) = 2, y'(\pi) = -1$$

$$\frac{dy}{dx} = y^4(x+1)^2, y(0) = -1$$



Linear Approximations

Approximate $\sin\left(\frac{\pi}{5}\right)$

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$



Taylor Polynomials

Find the 4th order Taylor Polynomial for $\ln(x)$ about $x_0=2$

$$\begin{aligned}f(x) &= f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ &\quad + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x - x_0)^4 + \dots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n.\end{aligned}$$



Inverse Trig Functions

Solve

$$\sin(\cos^{-1}(-1/3))$$



Exponents and Logs

Solve

$$\ln(x) + \ln(x-1) = 0$$

$$2 \log_3(x + 2) - \log_3 x = 2$$



Curve Sketching - Steps

- Examine $f(x)$ to determine
 - a. Domain
 - b. Asymptotes (Horizontal, Vertical and Oblique)
 - c. Symmetry (even and odd functions)
 - d. X and Y intercepts
- Take the first derivative to determine
 - a. Critical points
 - b. Where the derivative is undefined (singular points, end points, holes, vertical asymptote, etc)
 - c. Use a sign chart to determine where the function is increasing and decreasing
 - d. Classify points as local or global extrema
- Take the second derivative to determine
 - a. Any points where the second derivative is zero or undefined
 - b. Make a sign chart of these points to determine concavity
 - c. Identify inflection points (when the second derivative sign changes)
- Sketch the function



Questions that anyone has?

Otherwise, Good Luck!