Math 124 Midterm Tutorial

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What will be covered

- Polynomial Long Division
- Sums
- Riemann and Limit Definition
- Area of Plane Regions
- Definite Integrals
- FTC
- Inverse Trig Sub
- U Sub
- Integration by Parts
- Ration Function Integrals
- Improper Integrals

Tips

- Always differentiate to see if you get the integrand back or not
- Try a U-Sub quick, could save lots of time
- Don't forget +C !

Tricks for Rational

1. Factor first and see if anything cancels
2. Complete the square?
3. Separate numerator?
4. Polynomial long division
5. Partial Fractions

A Quick Recap on Polynomial Long Division

Divide

$$\frac{12x^3 - 11x^2 + 9x + 18}{4x + 3}$$

Sums

Find closed form value for

$$\sum_{i=1}^{n}(3i - i^2)$$

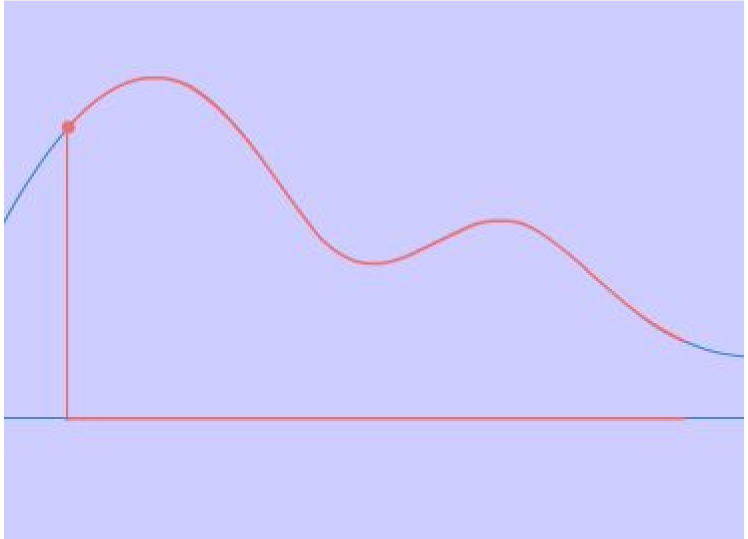
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

2017 Quiz 1

Riemann Sums



Draw a picture to see where the upper and lower portions are!

Riemann Sum

Evaluate the lower Riemann Sum of $f(x) = \sin x$ on $[0,\pi]$ with 4 equal subintervals

Definition of an Integral as a Limit of Sums

Evaluate using the definition of the definite integral as a limit of a sum.

$$\int_0^2 (1 - x^2)dx$$

2007 Midterm

Areas of Plane Regions

Find the unsigned area of the plane region bounded by the following curve

$y = 2^x$, and $y = x + 1$, where $0 \leq x \leq 1$

Definite Integrals

$$\int_0^{2\pi} |sinx|dx$$

$$\int_{-1}^{1} \frac{xe^{x^2}}{1+x^4+x^2} sin^2 x dx$$

2008 midterm

FTC (Leibniz integral rule)

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f(h(x))h'(x) - f(g(x))g'(x).$$

Find

$$\frac{d}{d\theta}\int_{\sin\theta}^{\cos\theta} \frac{1}{1-x^2}\,dx$$

Integrals

$$\int_0^4 x^3(x^2+1)^{-1/2}\,dx$$

Areas of plane regions

Find the area of the region bounded by $y = \tan x$, $y = -\tan x$, $x = 0$, and $x = \pi/4$.

Integrals

$$\int \frac{\sin^3 x}{\cos^6 x}\, dx$$

2008 M1

Average Value

$$\frac{1}{b-a}\int_a^b f(x)dx$$

What is the average value of $|\sin\theta - \cos\theta|$ over the interval $0 \leq \theta \leq \pi/2$?

http://wiki.ubc.ca/Science:Math_Exam_Resources/Courses/MATH101/April_2009/Question_01_(f)

Integrals

$$\int \frac{1}{x^2 + 6x + 13}\, dx$$

Rational Function Integrals

Remember...

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

$$\int \frac{3x^2+x+1}{(x^2+1)(x+1)}\,dx.$$

2016 Midterm

Rational Function Integrals

$$\int \frac{x - 3}{x^2 + 9}\, dx$$

2017 Quiz 5

Integrals

$$\int x \ln(1 + x)\, dx$$

Inverse Trig Sub

If you see	use the sub	so that	and
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$dx = a\cos\theta\,d\theta$	$\sqrt{a^2 - x^2} = a\cos\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2 + x^2} = a\sec\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$dx = a\sec\theta\tan\theta\,d\theta$	$\sqrt{x^2 - a^2} = a\tan\theta$

$$\int \frac{dx}{\sqrt{x^2+16}}$$

http://wiki.ubc.ca/Science:Math_Exam_Resources/Courses/MATH101/April_2008/Question_03_(d)

Harder Inverse Trig

$$\int \frac{\sqrt{x^2 - 4}}{x^3}\, dx$$

2017 Quiz 5

Integrals

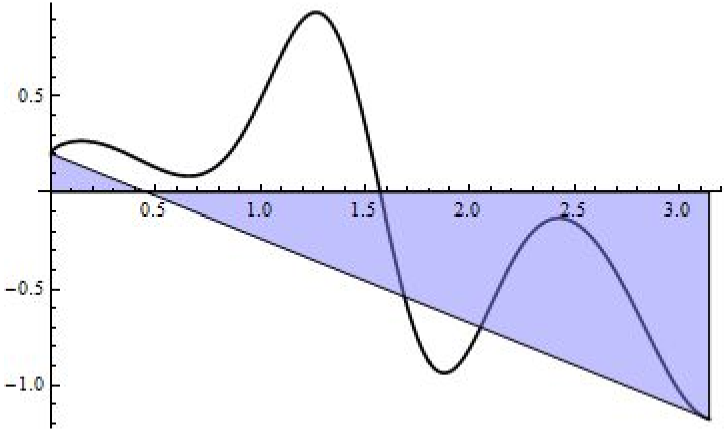
$$\int e^x \sin x \, dx$$

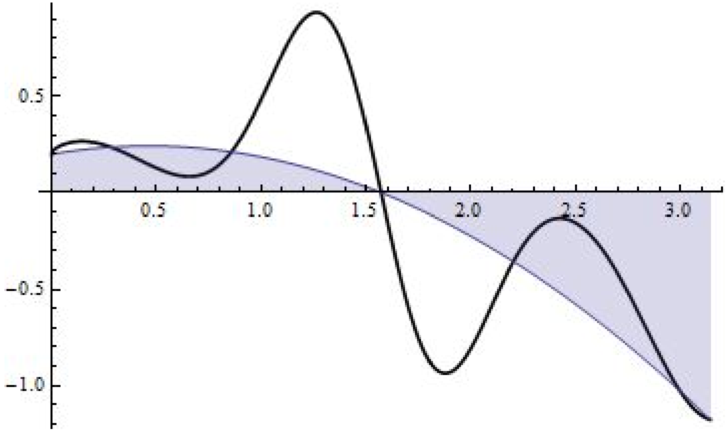
Improper Integrals

$$\int_0^\infty \frac{x}{1+2x^2}\,dx$$

6.5 Q12

Simpsons, Midpoint and Trapezoidal Rule





Formulae

Midpoint Rule

$$\int_a^b f(x)dx \approx M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x, \quad \bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$$

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

Simpson's Rule

$$\int_a^b y\,dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n)$$

- Simpson's rule needs odd number of data points

Bonus Problems

$$\int x\sec^2 x\,dx$$

$$\int (x^2+4)^{-3/2}\,dx$$

Good Luck!